IE170: Algorithms in Systems Engineering: Lecture 6

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Taking Stock

Last Time

- Divide-and-Conquer
- The Master-Theorem
- When the World Will End

This Time

- Master Theorem Practice
- Some Sorting Algs.
- Data Structures

The Master Theorem

- If recurrence has the form

$$T(n) = \begin{cases} \Theta(1) & n = 1 \\ aT(n/b) + f(n) & n > 1 \end{cases}$$

- The Master Theorem tells us how to analyze it:
  - If $f \in O(n^{\log_b a - \varepsilon})$, for some constant $\varepsilon > 0$, then $T \in \Theta(n^{\log_b a})$.
  - If $f \in \Theta(n^{\log_b a})$, then $T \in \Theta(n^{\log_b a} \lg n)$.
  - If $f \in \Omega(n^{\log_b a + \varepsilon})$, for some constant $\varepsilon > 0$, and if $af(n/b) \leq cf(n)$ for some constant $c < 1$ and $n > n_0$, then $T \in \Theta(f)$.

Some More Examples...

- Here we will do a couple examples of the master theorem
- Also I will show you a little trick (substitution) that can come in handy – especially if you have $\sqrt{\cdot}$

Not Fun!

- Homework 2.2-1: and prove that it has that form.
- Do all of 4.1

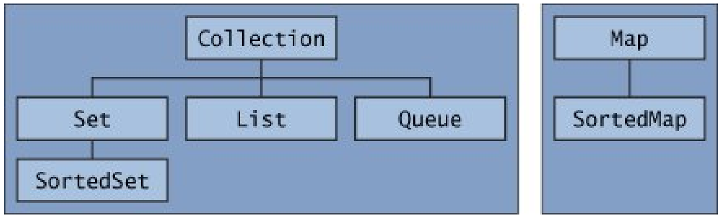
Fun!

Simple Sorting Algorithms:

- Merge Sort:
  - Divide the list into smaller pieces. Sort the small pieces. Then merge together sorted lists.
- Insertion Sort:
  - Insert item j into $A[0 \ldots j-1]$
- Selection Sort
  - Find j^{th} smallest element and put it in $A[j]$
- Bubble sort:
  - Start at end of array: If $A[j] < A[j-1]$, swap them

The Java Collections Interfaces

- In the remainder of the class, we will be using the Java Collections Interface: `http://java.sun.com/docs/books/tutorial/collections/TOC.html`
- Important: Most of what I will say only works if you set the "code level" to Java 5.0 in eclipse!
- The interfaces form a hierarchy:



(A subset of) the Collections Interface

```
public interface Collection<E> extends Iterable<E> {
    // Basic operations
    int size();
    boolean isEmpty();
    boolean contains(Object element);
    boolean add(E element);         //optional
    boolean remove(Object element); //optional
    Iterator<E> iterator();

    // Array operations
    Object[] toArray();
    <T> T[] toArray(T[] a);
}
```


Traversing Collections

- Use for-each construct:

```
for (Object o : collection)
    System.out.println(o);
```

- Use an iterator. (Can `remove()` items with iterator

```
for(Iterator<String> i= words.iterator(); i.hasNext(); ){
  System.out.println(i.next());
}
```

- `hasNext()`: returns true if the iteration has more elements,
- `next()`: returns the next element in the iteration.
- `remove()`: removes the last element that was returned by next from the underlying Collection.

Converting to Array

- Sometimes you need to convert a collection to an array:
  `Object[] a = c.toArray();`

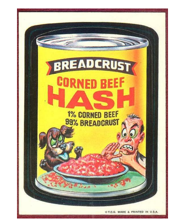
Set

- A Set is a Collection that cannot contain duplicate elements.
- It models the mathematical set abstraction.
- The Set interface contains only methods inherited from Collection and adds the restriction that duplicate elements are prohibited.
- Set is still an interface. There are 3 implementations of Set in Java.
  - HashSet
  - TreeSet
  - LinkedHashSet

Hash?

- No, Cheech. A hash table is a data structure in which we can "look up" (or search) for an element efficiently.
- The expected time to search for an element in a has table in $O(1)$. (Worst case time in $\Theta(n)$).
- Think of a hash table as an array
- With a regular array, we find the element whose "key" is j in position j of the array. `j = 17; val = a[j];` .
- This is called *direct addressing* and it takes $O(1)$ on your regular ol' random access computer.
- This form of direct addressing works when we can afford to have an array with one position for every possible key

More on Hash



- In a hash table the number of keys stored is small relative to the number of possible keys
- A hash table is an array. Given a key k, we don't use k as the index into the array – rather, we have a hash function h, and we use $h(k)$ as an index into the array.
- Given a "universe" of keys K.
  - Think of K as all the words in a dictionary, for example
- $h : K \to \{0, 1, \ldots m-1\}$, so that $h(k)$ gets mapped to an integer between 0 and $m-1$ for every $k \in K$
- We say that k hashes to $h(k)$

Example

- This look great. However, what happens if $h(k_1) = h(k_2)$ for $k_1 \neq k_2$?
- Two keys hash to the same value. The element collide
- This is typically handled by chaining
- Instead of storing a key k (or later key value pair (k, v)) at every position in the array, we store a linked list of keys.

A (Fairly) Obvious point

- BAD hash function. $h(k) = 3$.
- If all keys hash to the same value, then looking up a key takes $\Theta(n)$. (Since it is just a list).
- We would like a hash function to be "random" in the sense that a key k is equally likely to has into any of the m slots in the hash table (array).
- If have have such a function, then we can show that the time required to search for a key is $\Theta(1 + \frac{n}{m})$
- When hashing keys that are not numbers, you must convert them to numbers.

$$\text{BEER} = -142 + 2^4 + 5^3 + 5^2 + 18^1 = 42.$$

Back to the Java Collections

- So Now you know what a Java `HashSet` is.
- A `LinkedHashSet` is a `HashSet` that also keeps track of the order in which elements were inserted.
- (Think of laying a linked list on top of the Hash Table)
- A `TreeSet` stores its elements in a alertred-black tree.
- In order to understand red-black trees, we must know about binary search trees.
- Hash table is "good" at INSERT(), SEARCH(), DELETE(). But what if you also want to support (efficiently) MINIMUM(), MAXIMUM()

Binary Search Tree

- A binary search tree is a data structrue that is conceptualized as a binary tree. (Have you read Appendix B-4 yet?)
- Each node in the tree contains:
  - key k. (Or maybe (key, value): (k, v))
  - left l: Points to the left child
  - right r: Points to the right child
  - parent p: Points to the parent

Binary Search Tree Property

If y is in the left subtree of x, then $k(y) \leq k(x)$

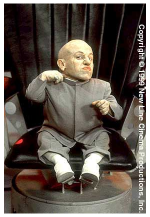
Binary Search Trees

- There are lots of binary trees that can satisfy this property.
- It is obvious that the number of binary tree on n nodes b_n is

$$b_n = \frac{1}{n+1}\binom{2n}{n} \qquad b_n = \frac{4^n}{\sqrt{\pi}n^{3/2}}(1 + O(1/n))$$

- And not all of these (exponentially many) are created equal.
- In fact, we would like to keep our binary search trees "short", because most of the operations we would like to support are a function of the height h of the tree.

Short Is Beautiful



- SEARCH() takes $O(h)$
- MINIMUM(), MAXIMUM() also take $O(h)$
- Slightly less obvious is that INSERT(), DELETE() also take $O(h)$
- Thus we would like to keep out binary search trees "short" (h is small).

red-black Trees

- red-black trees are simply a way to keep binary search trees short. (Or balanced)
- Balanced here means that no path on the tree is more than twice as long as another path.
- An implication of this is that its maximum height is $2\lg(n+1)$
- SEARCH(), MINIMUM(), MAXIMUM(), all take $O(\lg n)$
- It's implementation is complicated, so we won't cover it
- INSERT(): also runs in $O\lg(n)$
- DELETE(): runs in $O\lg(n)$
  - (but it is more complicated to maintain the "red-black" property)

Next Time?

- More on data structures and Java collections
- The greatest lab ever

Small News

Let's have a LITTLE quiz on 2/7