IE 495 – Stochastic Programming
Problem Set #1

Due Date: February 3, 2003

Do any four of the following five problems. If you are working in pairs, do all five problems. You are allowed to examine outside sources, but you must cite any references that you use. Please don't discuss the problems with other members of the class (other than your partner, if you are working with one).

1 Random Linear Programs and the Distribution Problem

Recall the random linear program that we saw in class:

minimize

$$x_1 + x_2$$

subject to

$$\begin{aligned} \omega_1 x_1 + x_2 &\geq 7 \\ \omega_2 x_1 + x_2 &\geq 4 \\ x_1 &\geq 0 \\ x_2 &\geq 0 \end{aligned}$$

with $\omega_1 \sim \mathcal{U}[1, 4]$ and $\omega_2 \sim \mathcal{U}[1/3, 1]$.
Let

- $(x_1^*(\omega), x_2^*(\omega))$ be the optimal solution for a given value of $\omega = (\omega_1, \omega_2)$.
- $v^*(\omega) = x_1^*(\omega) + x_2^*(\omega)$ be the optimal objective function value.

1.1 Problem
Calculate $x_1^*(\omega), x_2^*(\omega)$, and $v^*(\omega)$ for all $\omega \in \Omega = [1, 4] \times [1/3, 1]$.

1.2 Problem
Calculate the distribution function F_{v^*} of the random variable $v^*(\omega)$.

1.3 Problem
Calculate the density of $v^*(\omega)$.

1.4 Problem
Calculate $\mathbb{E}_\omega[v^*(\omega)]$.

2 Proofs

To answer these questions, you will need to know the following definitions.

1 Definition
A function $f : D(f) \mapsto R(f)$ is *Lipschitz continuous* on its domain $D(f)$ if and only if there exists a constant L (called the Lipschitz constant) such that

$$|f(a) - f(b)| \leq L|a - b|$$

for all $a, b \in D(f)$

Note: The definition of Lipschitz continuity on page 78 of Birge and Louveaux is just plain wrong.

2 Definition
A set $S \subseteq \Re^n$ is polyhedral if and only if it can be written as

$$S = \{x \in \Re^n : Ax \leq b\}$$

for a matrix $A \in \Re^{m \times n}$ and vector $b \in \Re^m$.

For each of the problems, state whether they are true or false. In either case, provide a proof. (If the statement is false, then a counterexample is such a proof).

2.1 Problem
The intersection of convex sets is convex.

2.2 Problem
The union of convex sets is convex

2.3 Problem
All convex functions are Lipschitz continuous on their domain.

2.4 Problem
All polyhedral sets are convex.

2.5 Problem
Let $f : \Re^n \mapsto \Re$ be a convex function. Then for any $x_1, x_2, \ldots, x_k$, and all sets of "convex multipliers" $\lambda_1, \lambda_2, \ldots, \lambda_k$ such that $\sum_{i=1}^k \lambda_i = 1$ and $\lambda_i \geq 0 \ \forall i = 1, 2, \ldots, k$,

$$f(\sum_{i=1}^{k} \lambda_i x_i) \leq \sum_{i=1}^{k} \lambda_i f(x_i).$$

3 Farmer Ted

3.1 Problem
Birge and Louveaux #1.3, Page 18.

3.2 Problem
Birge and Louveaux #1.5, Page 18.

3.3 Problem
Birge and Louveaux #1.7, Page 19.

4 Yield Management

For the problem described by Birge and Louveaux, Page 43, Number 1...

4.1 Problem
Develop the stochastic programming model.

4.2 Problem
Solve the stochastic program.

4.3 Problem
Calculate the value of the stochastic solution (VSS).

5 Hospital Staffing

For the problem described by Birge and Louveaux, Page 46, Number 6[1]...

5.1 Problem
Develop a resonable stochastic programming model.

5.2 Problem
Solve the stochastic program.

5.3 Problem
Calculate the value of the stochastic solution (VSS).

[1]Note – I believe there is an error in the book. When solving this problem, use (100, 90, *120*) as the first of the 9 demand scenarios