

IE 495 – Stochastic Programming

Problem Set #2

Due Date: February 26, 2003

Do all three of the following problems. If you work alone, you will receive a 10% bonus on your score. You are allowed to examine outside sources, but you must cite any references that you use. Please don't discuss the problems with other members of the class (other than your partner, if you are working with one).

1 Math Time

1.1 Problem

Birge and Louveaux #4, Page 102.

Interlude

You will need to know the following stuff to answer problem #2.

1 Definition

A function $f : D(f) \mapsto R(f)$ is *Lipschitz continuous* on its domain $D(f)$ if and only if there exists a constant L (called the Lipschitz constant) such that

$$|f(a) - f(b)| \leq L|a - b|$$

for all $a, b \in D(f)$

2 Definition

The *left-derivative* of a function $f : \Re \mapsto \Re$ is the quantity

$$f'_-(x) = \lim_{x \rightarrow t_-} \frac{f(x) - f(t)}{x - t} = \sup_{x < t} \frac{f(x) - f(t)}{x - t}.$$

3 Definition

The *right-derivative* of a function $f : \Re \mapsto \Re$ is the quantity

$$f'_+(x) = \lim_{x \rightarrow t_+} \frac{f(x) - f(t)}{x - t} = \inf_{x > t} \frac{f(x) - f(t)}{x - t}.$$

1 Theorem

If $f : \Re \mapsto \Re$ is a (proper) convex function, then $\partial f(x) = [f'_-(x), f'_+(x)]$.

2 Shortage Properties

Let ω be a one-dimensional random variable with cumulative distribution F . Define the *expected shortage* function as

$$H(x) = \mathbb{E}_\omega [(\omega - x)^-].$$

Assume that $\mu = \mathbb{E}_\omega [(\omega)^-] < \infty$.

2.1 Problem

Prove $H(x)$ is convex.

2.2 Problem

Prove $H(x)$ is Lipschitz continuous. What is the Lipschitz constant?

2.3 Problem

What is the left derivative of H at x ?

2.4 Problem

What is the right derivative of H at x ?

2.5 Problem

What is $\partial H(x)$?

3 Network Design

This problem is concerned with a simple network planning model. The goal of the planning model is to decide how to allocate (limited) additional capacity resources in order to meet a forecast demand. We would like to minimize the expected number of unserved requests for services while satisfying the budgetary limitations on the total capacity expansion.

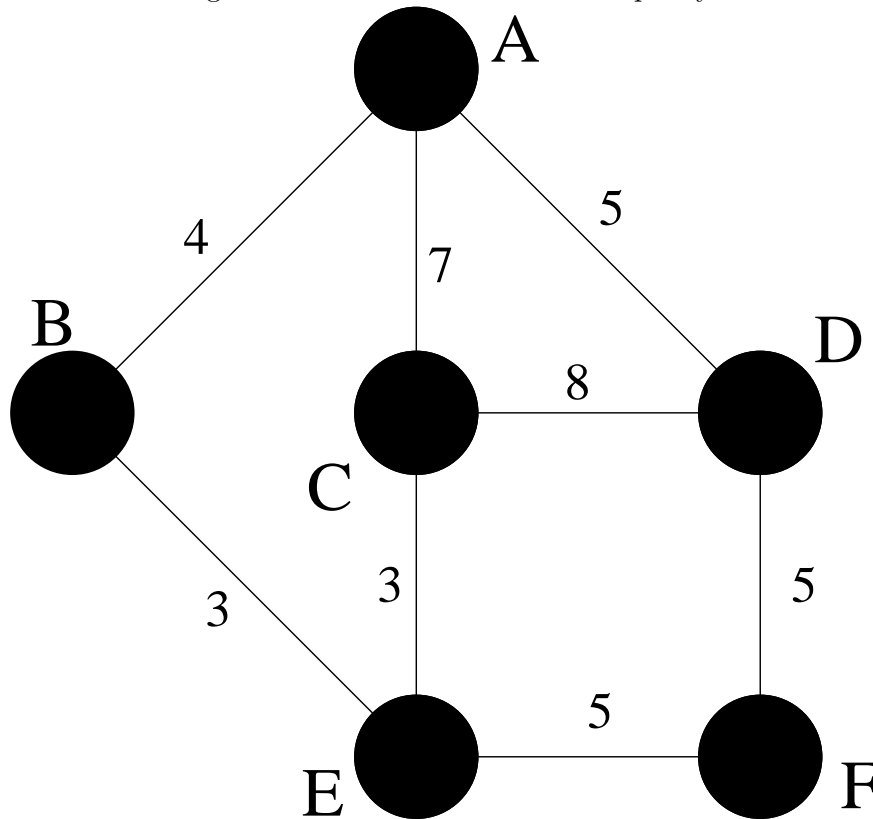
Specifically, consider the network shown in Figure 1. The installed capacities are shown on the figure and also in Table 2. We are given the point-to-point demands d shown in Table 1. The cost of installing one unit of additional capacity is also given in Table 2. You can install fractional capacity, and you may not install any new links in the network.

Point-to-Point Pair	Demand
A-E	8
B-D	8
C-F	4
B-F	6
A-F	4

Table 1: Demand between nodes

For each point to point pair in Table 1, there are a set of feasible routes that can carry demand between the pair. These feasible routes are listed in Table 3. There is a budget of 30 to install new capacity.

Figure 1: Network and Installed Capacity



Link	Installed Capacity	Cost for New Capacity
AB	4	2
AC	7	3
AD	5	2
BE	3	4
CD	8	3
CE	3	1
DF	5	2
EF	5	2

Table 2: Capacity on Links and Cost for Additional Capacity

Demand	Routes
A-E	ABE, ACE, ADCE, ADFE
B-D	BAD, BACD, BECD, BEFD
C-F	CDF, CEF
B-F	BADF, BEF
A-F	ADF, ACDF, ACEF

Table 3: Feasible Routes

3.1 Problem

Formulate a linear program that decides how much additional capacity to install on each arc in a way that minimizes the total amount of unserved demand subject to the budgetary restriction.

Hint: You will have variables x_j for additional capacity, and f_{ir} for the amount of demand i that flows on route r

3.2 Problem

Create a valid MPS file representing your formulation in Problem 3.1, and email it to jt13@lehigh.edu.

3.3 Problem

Solve your formulation from Problem 3.1.

3.4 Problem

Unfortunately, the demands in Table 1 are random. Suppose there are demand scenarios $d_s, s = 1, 2, \dots, S$ each occurring with probability p_s . Formulate a stochastic programming version of Problem 3.1 that will minimize the *expected* amount of unserved demand.

3.5 Problem

Let $d \in \Re^5$ be the demand in Table 1. Suppose that there are three demand scenarios $d_1 = 0.75d, d_2 = d, d_3 = 1.25d$, each occurring with probability $1/3$. Create the proper SMPS files for this instance and mail them to jt13@lehigh.edu.

3.6 Problem

Solve your formulation from Problem 3.5. What is the value of the stochastic solution in this case?

3.7 Problem**(Challenge – Bonus)**

Now each of the point-to-point demands *independently* varies. For each point-to-point pair i the demand varies as $d_i \approx \mathcal{U}(0, 3d_i)$. Solve this instance as accurately as you can. The people with the two most accurate solutions win *FREE LUNCH* with me.