Stochastic Programming

IE495

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Today's Outline

- About this class.
- About me
- Say Cheese
- Quiz Number 0
- Why should we care about stochastic programming?
- Yucky math review

Class Overview

- Meeting Times: Monday-Wednesday 4:10–5:30
- Office Hours: (*Please* try to use them).
  - Monday 5:30-6:30PM
  - Wednesday 5:30-6:30PM
  - Thursday 2-4PM
  - By Appointment (8-4879)
- Course HomePage:
  - `http://www.lehigh.edu/~jtl3/teaching/ie495`
  - I will try to post (draft) outlines of lecture notes there before class.
- Syllabus dates are somewhat tentative

Course Details

- Learning is better if you participate.
  - ◇ I will call on you during class. (Gasp!)
- Seven(?) Problem Sets
  - ◇ I will throw out the lowest score when computing the average at the end.
  - ◇ Don't be late! 10% Grade penalty for every late day.
- Final Exam
  - ◇ Take home
- Class project...

The Project

- A significant portion of this class will be an individual project
  - Everyone should aim to have their project decided on by the beginning of next month.
  - I will (probably) have you create a short project proposal outlining what you intend to do

Project Ideas

- ⋆ Implementation-based[a]
  - ◇ I have a long list of potential projects listed in the syllabus.
  - ◇ Incorporate stochastic programming modeling into your current line of research
- Paper survey
  - ◇ Read and report on *three* separate papers in a chosen area of stochastic programming.
  - ◇ I will develop a bibliography of some suggested papers.
- Please arrange a time to contact me if you have questions about the project.

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[a] Preferred Project Type

Grading

- ⋆ I don't view grades in (elective) graduate courses as very important.
- You should be here because you want to be here, and you should learn because you want to learn.
- Nevertheless, they make me assign grades. Therefore...
  - ◇ 25% Project
  - ◇ 50% Problem Sets
  - ◇ 25% Final Exam

Course Topics (Subject to Change)

- Modeling
  - A little math background
  - “Stages” and “recourse”
  - Formulating the deterministic equivalent (DE) of a stochastic program
  - Formulating and solving (DE)’s with an AML
  - Examples
- Theory – Recourse problems
  - Two-stage stochastic LP
  - Multi-stage stochastic LP
  - Stochastic IP

More Course Topics

- Theory – Probabilistic Constraints
- Algorithms (mostly for solving recourse problems)
- Sampling
- Applications and Cutting Edge Research

⋆ I will introduce mathematical concepts and computational tools as needed.

Course Objectives

- Learn the terms, basic capabilities, and limitations of stochastic programming models.
- Learn to formulate analytical models with quantified uncertainty as stochastic programs
- Learn the basic theory required to understand the structure of stochastic programs
- Learn the algorithmic techniques used to solve stochastic programs
- Learn new computational tools

Objectives

Accomplishing these objectives, you will be able to...

- Incorporate stochastic programming techniques into your current research projects
- Develop state-of-the-art software and algorithms for stochastic programs
- Familiarize yourselves with the "state-of-the-art" in stochastic programming by reading and understanding recent technical papers

Great Expectations

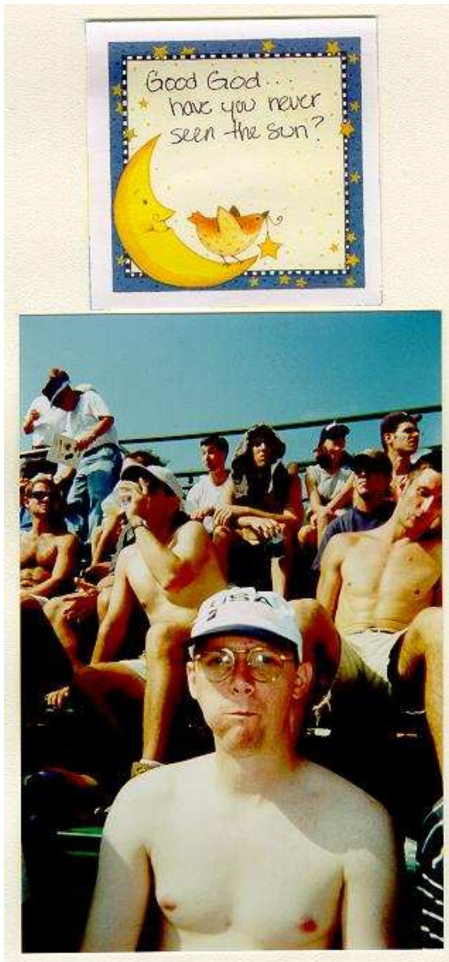
I am expected to...

- Teach
- Be at my office hours
- Give you feedback on how you are doing in a timely fashion

You are expected to...

- Learn
- Attend lectures and participate
- Do the problem sets
- Not be rude, if possible.
  - Sleeping, Cell Phones, Leaving in the middle of lecture

About me...



- B.S. (G.E.), UIUC, 1992.
- M.S., OR, GA Tech, 1994.
- Ph.D., Optimization, GA Tech, 1998
- 1998-2000 : MCS, ANL
- 2000-2002 : Axioma, Inc.
- Research Areas: Large Scale Optimization, High Performance Computing.
- Married. One child, Jacob, born 10/28. He is awesome.
- Hobbies: Golf, Integer Programming.

Picture Time

Stochastic Programming

? What does "Programming" mean in "Mathematical Programming", "Linear Programming", etc...?

- Mathematical Programming (Optimization) is about decision making.
- Stochastic Programming is about decision making *under uncertainty*.
- View it as "Mathematical Programming with random parameters"

Dealing With Randomness

- Typically, randomness is ignored, or it is dealt with by
  - ◇ Sensitivity analysis
    - For large-scale problems, sensitivity analysis is useless
  - ◇ "Careful" determination of instance parameters
    - No matter how careful you are, you can't get rid of inherent randomness.

⋆ Stochastic Programming is the way!

Stochastic Programming

- ⋆ Fundamental assumption : We know a (joint) probability distribution.
- This may seem limiting, but...
  - ◇ You may not need to know the whole joint distribution. (You generally only care about the impact of randomness on some random variables).
  - ◇ A "subjective" specification of the joint distribution can give useful information
    - ⋆ Probably the (deterministic) problem has parameters people would consider subjective
- If you *really* don't known anything about the probability, you can try a "fuzzy" approach.

Types of Uncertainty

Where does uncertainty come from?

- Weather Related
- Financial Uncertainty
- Market Related Uncertainty
- Competition
- Technology Related
- Acts of God

- In an analysis of a decision, we would proceed through this list and identify those items that might interact with our decision *in a meaningful way!*

The Scenario Approach

- A scenario-based approach is by no means the only approach to dealing with randomness, but it does seem to be a reasonable one.
- The scenario approach assumes that there are a finite number of decisions that nature can make (outcomes of randomness). Each of these possible decisions is called a *scenario*.

Ex. Demand for a product is "low, medium, or high".

Ex. Weather is "dry" or "wet".

Ex. The market will go "up" or "down"

⋆ Even if the nature acts in a continuous manner, often a discrete approximation is useful.

A First Example

- Farmer Fred can plant his land with either corn, wheat, or beans.
- For simplicity, assume that the season will either be wet or dry – nothing in between.
- If it is wet, corn is the most profitable
- If it is dry, wheat is the most profitable.

Profit

	All Corn	All Wheat	All Beans
Wet	100	70	80
Dry	-10	40	35

- So if the probability of a wet season is p. The expected profit of planting the different crops is
  - Corn: $-10 + 110p$
  - Wheat: $40 + 30p$
  - Beans: $35 + 45p$

What's the Answer?

- Suppose $p = 0.5$, can anyone suggest a planting plan?
- Plant 1/2 corn, 1/2 wheat?
  - Expected Profit: 0.5 (-10 + 110(0.5)) + 0.5 (40 + 30(0.5)) = 50

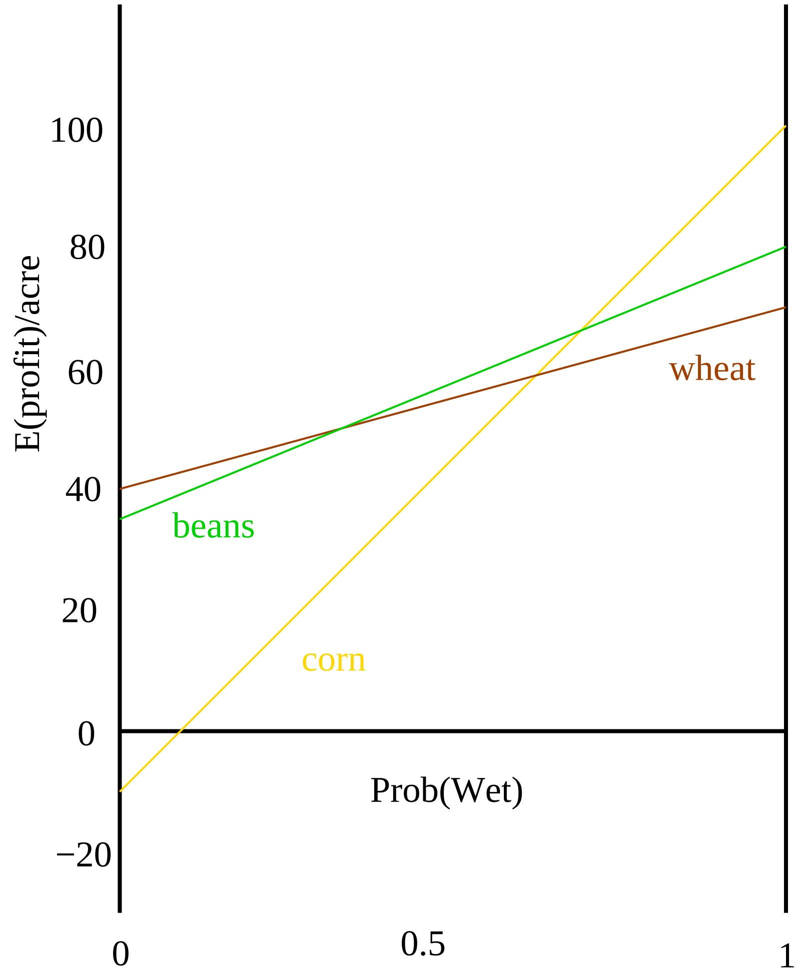
? Is this optimal?

No!

- Suppose $p = 0.5$, can anyone suggest a planting plan?
- Plant all beans!
  - ◇ Expected Profit: $35 + 45(0.5) = 57.5$!

⋆ The expected profit in behaving optimally is 15% better than in behaving reasonably.

Profit Picture



What Did We Learn

⋆ Averaging Solutions Doesn't Work!

- The best decision for today, when faced with a number of different outcomes for the future, is in general not equal to the "average" of the decisions that would be best for each specific future outcome.

- That example is a little too simplistic for us to draw too many conclusions other than you cannot just average solutions.
- You *can't* replace random parameters by their mean value and solve the problem. This is (in general) not optimal either!

Probability Stuff

- Stochastic programming is like linear programming with "random" parameters.

⇒ It makes sense to do just a bit of review of probability.

- ω is an "outcome" of a random experiment.
- The set of all possible outcomes if Ω.
- The outcomes can be combined into subsets $\mathcal{A}$ of Ω (called events).

Probability spaces

- For each $A \in \mathcal{A}$ there is a probability measure (or distribution) P that tells the probability with which $A \in \mathcal{A}$ occurs.
  - ◇ $0 \leq P(A) \leq 1$
  - ◇ $P(\Omega) = 1$, $P(\emptyset) = 0$
  - ◇ $P(A_1 \cup A_2) = P(A_1) + P(A_2)$ if $A_1 \cap A_2 = \emptyset$.

⋆ The triple $(\Omega, \mathcal{A}, P)$ is called a *probability space*.

Random Variable

- A random variable ξ on a probability space $(\Omega, \mathcal{A}, P)$ is a real-valued function $\xi(\omega), (\omega \in \Omega)$ such that $\{\omega | \xi(\omega) \le x\}$ is an event for all finite x.
  - ◇ So $(\xi \le x)$ is an event, and can be assigned a probability.
- ξ has a *cumulative distribution* given by $F_\xi(x) = P(\xi \le x)$.
- *Discrete* random variables take on a finite number of values $\xi^k, k \in K$
  - ◇ Density: $f(\xi^k) \equiv P(\xi = \xi^k)$ $\qquad (\sum_{k \in K} f(\xi^k) = 1)$.
- Continuous random variables have density $f(\xi)$.
  - ◇ $P(\xi = x) = 0$
  - ◇ The probability of ξ being in an interval $[a, b]$ is...

More

$$
\begin{aligned}
P(a \le \xi \le b) &= \int_a^b f(\xi) d\xi \\
&= \int_a^b dF(\xi) \\
&= F(b) - F(a)
\end{aligned}
$$

- *Expected value* of ξ is
  - $\mathbb{E}(\xi) = \sum_{k \in K} \xi^k f(\xi^k)$ (Discrete)
  - $\mathbb{E}(\xi) = \int_{-\infty}^{\infty} f(\xi) d\xi = \int_{-\infty}^{\infty} dF(\xi)$.
- *Variance* of ξ is $\mathrm{Var}(\xi) = \mathbb{E}(\xi - \mathbb{E}(\xi)^2)$.

Next Time

- Modeling, Modeling, Modeling.
  - Stages and Recourse
  - Farmer Ted
  - Using AMPL